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Fokker-Planck approach to quantum transport statistics: Calculation of limiting probability distributions

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We discuss the quantum-mechanical scattering of electrons from a disordered quasi-one-dimensional wire in terms of its transfer matrix $\mathbf{T}$. An alternative method for the determination of the limiting probability distributions of the eigenvalues and eigenvectors of $\ln \mathbf{T}\mathbf{T}^\dagger$ is presented. It generalizes a previous approach which was successfully applied to the one-dimensional limit. We show rigorously that the orientations of the system of eigenvectors of equivalent-channel models which were discussed in earlier work are uniformly distributed.

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Substantial progress in the analytical understanding of electron scattering from disordered samples connected to ideal multichannel leads has been made [1–4] during the past few years. Nowadays this field receives widespread interest since it is closely related to multiple scattering of classical waves in random media [5] and also because of its importance for the theory of quantum chaos [6]. Solid-state physicists became interested in the problem when Landauer discovered [7] that the zero temperature resistance of a finite one-dimensional (1D) disordered system can be expressed in terms of its quantum-mechanical transmission properties. Various authors [8] extended the idea to quasi-1D wires with many channels. The scattering approach was applied in numerical finite-size scaling analyses of the Anderson transition [9] and used to argue [10] in favor of the one-parameter scaling theory [11]. Weller and Kasner [12] combined it with Berezinskii's diagrammatic method and constructed a recursion scheme for the calculation of the localization length of coupled disordered chains. Pendry and co-workers obtained various results [13] by expressing the transmission properties in terms of the transfer matrix $\mathbf{T}$.

The transfer matrix transforms the amplitudes of the propagating wave modes at the Fermi energy (channels) on the left side of the sample into the amplitudes on the right side. Thus, the transfer matrix of two samples joined together is the product of the transfer matrices of each sample. For a long disordered sample consisting of thin slices, $\mathbf{T}$ can be obtained by multiplying a large number of random matrices. Without spin-orbit scattering and without a magnetic field, $\mathbf{T}$ can be written in a polar decomposition which respects time reversal invariance and flux conservation [14,15]

$$\mathbf{T} = \begin{pmatrix} \mathbf{u} & \mathbf{0} \\ \mathbf{0} & \mathbf{u}^* \end{pmatrix} \begin{pmatrix} \sqrt{1+\boldsymbol{\lambda}} & \sqrt{\boldsymbol{\lambda}} \\ \sqrt{\boldsymbol{\lambda}} & \sqrt{1+\boldsymbol{\lambda}} \end{pmatrix} \begin{pmatrix} \mathbf{v} & \mathbf{0} \\ \mathbf{0} & \mathbf{v}^* \end{pmatrix} \quad (1)$$

where $\mathbf{u}$ and $\mathbf{v}$ are unitary $N \times N$ matrices and λ is diagonal, real, and positive. The eigenvalues of $\mathbf{T}\mathbf{T}^\dagger$ are $\exp \pm\Gamma_j$ where $\cosh\Gamma_j = 1 + 2\lambda_j$. The corresponding eigenvectors are essentially the columns of $\mathbf{u}$. From theorems on random matrix products [16,17] one expects that $\mathbf{u}$ and $\mathbf{v}$ have stationary limiting probability distributions for large system lengths L and that $\alpha_j \equiv \Gamma_j/2L$ are self-averaging. $\alpha_j^\infty := \lim_{L\to\infty} \alpha_j(L)$ are the Lyapunov exponents of $\mathbf{T}$. They characterize the exponential growth of λ_j with L. The two-terminal conductance for spinless electrons in units of e^2/h is given by [18,19] $g = \sum_{j=1}^N 2/(1 + \cosh\Gamma_j)$. Since $g \approx \exp(-2\alpha_1^\infty L)$ for large L, the smallest Lyapunov exponent α_1^∞ is identified with the inverse localization length $1/\xi$.

For continuous models one can derive a Fokker-Planck equation (FPE) for the evolution of the probability distribution of $\mathbf{T}$ with L, as shown in a pioneering paper by Dorokhov [14]. He investigated a microscopic model of N weakly coupled chains and calculated the N dependence of the localization length. The validity of his results was limited to weakly disordered samples with a cross-sectional diameter of less than the mean free path. This was due to an average over the L dependence of the drift and diffusion coefficients which oscillate rapidly with L in such samples. The recent discovery of the universal conductance fluctuations in metallic mesoscopic systems [20] led Mello, Pereyra, and Kumar [1] to design a model which was amenable to a rigorous analytical evaluation of quantities like the mean and the variance of the conductance in the metallic regime. They specified the probability distribution of a thin slice by maximizing its entropy under the constraint that the conductance obey Ohm's law. As a consequence, $\mathbf{u}$ and $\mathbf{v}$ were uniformly distributed on the unitary group for all system lengths and a closed FPE for $\boldsymbol{\lambda}$ could be derived. There were no correlations between $\boldsymbol{\lambda}$, $\mathbf{u}$, and $\mathbf{v}$. It has been shown that this "isotropic model" (IM) is intimately related to a global maximum entropy approach on the space of the transfer matrices [21]. In the sequel, the IM was generalized in order to include a magnetic field and spin-orbit scattering [22–24]. An exact solution of the magnetic-field case is now available [4]. All of the obtained results are in precise agreement with those of microscopic models for metallic quasi-1D samples [20,2]. However, 2D and 3D samples cannot be described within the IM because the maximum entropy approach eliminates all the information about the transverse structure of the sample. In order to overcome this difficulty Mello and Tomsovic [25] introduced a more general phenomenological model which takes the transverse structure into account. Recently Chalker and Bernhardt [26] considered a special case, namely weakly coupled wires. They expressed the Lyapunov exponents of $\mathbf{TT}^\dagger$ in terms of the large-L limiting probability distribution of $\mathbf{u}$ and constructed an approximation scheme to calculate relevant features of this distribution. This allowed them to study the Anderson transition within their model.

In the present paper we will show that the oscillations of the drift and diffusion coefficients can be treated in a systematic way which extends the range of validity of the FPE to samples with arbitrary transverse extension and disorder. The expression for the Lyapunov exponents in [26] is recovered. In addition, we express all the moments of Γ_j in terms of the large-L limiting probability distribution of $\mathbf{u}$. The latter is the stationary solution to a FPE which is explicitly derived. An alternative method which will allow us to determine the stationary solution for more than two wires is presented. Here we use it to show rigorously that the stationary solution of equivalent-channel models (ECM's), which were discussed in an earlier work, is the uniform distribution on the unitary group.

We consider disordered 1D wires which are coupled by random hopping matrix elements,

$$H_{nn'} = -\delta_{nn'}\frac{\hbar^2}{2m}\partial_x^2 + V_{nn'}(x), \tag{2}$$

where $V_{nn'}(x)$ is real and symmetric in its indices. $V_{nn'}(x)$ is zero in the leads and stochastic for $x \in [0, L]$. It describes on-wire disorder and/or random hopping between the wires. The randomness is assumed to be Gaussian white noise with zero average

$$\langle V_{nn'}(x)V_{mm'}(x')\rangle$$
$$= U_{nn'}\delta(x-x')(\delta_{nm}\delta_{n'm'} + \delta_{nm'}\delta_{n'm}). \tag{3}$$

The inverse mean free paths for backward and forward scattering from wire n into wire n' are defined by $1/l_{nn'} := \lim_{\delta L\to 0}|r_{nn'}|^2/\delta L$ and $1/l'_{nn'} := \lim_{\delta L\to 0}|t_{nn'}|^2/\delta L$, respectively. $r_{nn'}$ and $t_{nn'}$ are the reflection and transmission amplitudes of a thin slice of length δL, respectively. Here, they are given by $1/l_{nn'} = 1/l'_{nn'} = (m/\hbar^2 k)^2 U_{nn'}(1+\delta_{nn'})$ where $E = (\hbar k)^2/2m$ is the energy of the scattered electron.

In order to eliminate as many coordinates as possible, we follow Dorokhov [14] and study $\mathbf{M} = \mathbf{TT}^\dagger$, which eliminates $\mathbf{v}$. Assume that the probability distribution $\bar{p}(L;\mathbf{M})$ of the matrix $\mathbf{M}$ for a system of length L is known and a thin slice of length δL is added. Since the disorder is uncorrelated, one can derive the convolution property [27]

$$\bar{p}(L+\delta L;\mathbf{M}) = \langle \bar{p}(L;\mathbf{T}_{\delta L}^{-1}\mathbf{M}(\mathbf{T}_{\delta L}^\dagger)^{-1})\rangle_{(L,\delta L)}, \tag{4}$$

where $\langle\ \rangle_{(L,\delta L)}$ is the disorder average over the transfer matrix $\mathbf{T}_{\delta L}$ of the slice. The measure of $\bar{p}(L;\mathbf{M})$ is

$$d\rho(M) = J(\mathbf{\Gamma})\prod_{i=1}^{N} d\Gamma_i d\mu(\mathbf{u}), \tag{5}$$

where $J(\mathbf{\Gamma}) = \prod_{i<j}|\cosh\Gamma_i - \cosh\Gamma_j|\prod_i \sinh\Gamma_i$. The measure is invariant under the transformation $\mathbf{TMT}^\dagger$ for any $\mathbf{T}$ [27]. $d\mu(\mathbf{u})$ is the invariant measure on the unitary group. Equation (4) is the starting point for the derivation of the FPE of $\bar{p}$. We want to treat the large-L limit of the probability distribution $p(L;\mathbf{\Gamma},\mathbf{u},\mathbf{u}^*) := J(\mathbf{\Gamma})\bar{p}(L;\mathbf{M})$. This leads to a considerable simplification [28,14]. Due to the self-averaging of α_j one expects that Γ_j can be ordered $1 \ll \Gamma_1 \ll \Gamma_2 \ll \cdots \ll \Gamma_N$ if $L \gg \xi = 1/\alpha_1^\infty$. Therefore, we approximate $\coth\Gamma_j \approx 1$, $\sinh\Gamma_i/(\cosh\Gamma_i - \cosh\Gamma_k) \approx 1$ if $i > k$ or ≈ 0 if $i < k$ and extend the range of all Γ_i to $-\infty$ [28]. Then all except one selected Γ_j may be integrated out. The FPE for the resulting probability distribution $p_j(L;\Gamma_j,\mathbf{u},\mathbf{u}^*)$

$$\frac{\partial p_j}{\partial L} = (\hat{A}_j\partial_{\Gamma_j}^2 + \hat{B}_j\partial_{\Gamma_j} + \hat{C})p_j \tag{6}$$

correctly describes the bulk of the probability distribution of Γ_j for large L. The operators $\hat{A}_j$, $\hat{B}_j$, and $\hat{C}$ acting on $\mathbf{u}$ and $\mathbf{u}^*$ are [27]

$$\begin{aligned}\hat{A}_j &= \frac{1}{2}[\Delta\Gamma_j^2],\\ \hat{B}_j &= -[\Delta\Gamma_j] + \partial_{u_{mn}}[\Delta\Gamma_j\Delta u_{mn}] + \partial_{u^*_{mn}}[\Delta\Gamma_j\Delta u^*_{mn}],\\ \hat{C} &= -\partial_{u_{mn}}[\Delta u_{mn}] - \partial_{u^*_{mn}}[\Delta u^*_{mn}]\\ &\quad + \partial_{u_{mn}}\partial_{u^*_{m'n'}}[\Delta u_{mn}\Delta u^*_{m'n'}]\\ &\quad + \frac{1}{2}\partial_{u_{mn}}\partial_{u_{m'n'}}[\Delta u_{mn}\Delta u_{m'n'}]\\ &\quad + \frac{1}{2}\partial_{u^*_{mn}}\partial_{u^*_{m'n'}}[\Delta u^*_{mn}\Delta u^*_{m'n'}],\end{aligned} \tag{7}$$

where a summation over repeated indices is understood. $\Delta\Gamma$, $\Delta\mathbf{u}$, and $\Delta\mathbf{u}^*$ are the changes of the parameters of $\mathbf{M}$ which are induced by the transformation $\mathbf{T}_{\delta L}^{-1}\mathbf{M}(\mathbf{T}_{\delta L}^\dagger)^{-1}$, and $[\cdots] = \lim_{\delta L\to 0}(1/\delta L)\langle\cdots\rangle_{(L,\delta L)}$. Note that the derivatives in Eq. (7) treat u_{mn} and u^*_{mn} as independent without taking the unitarity of the matrix $\mathbf{u}$ into account. But since the FP operator commutes with the delta function $\delta(\mathbf{1}-\mathbf{uu}^\dagger)$ [27], unitarity is conserved . The terms in the brackets can be obtained by second-order perturbation theory or by iteration of the Langevin equations for $\mathbf{\Gamma}$ and $\mathbf{u}$. For the derivation of the Langevin equations $\mathbf{T}_{\delta L}$ is needed to first order in δL,

$$\mathbf{T}_{\delta L} = \mathbf{1} + \begin{pmatrix}\gamma & \eta\\ \eta^* & \gamma^*\end{pmatrix}\delta L + O(\delta L^2), \tag{8}$$

where $\gamma_{nn'} = -im/\hbar^2 k V_{nn'}(L)$, $\eta_{nn'} = \exp(-i2kL)\gamma_{nn'}$. The operators $\hat{A}_j$, $\hat{B}_j$, and $\hat{C}$ do not depend on Γ_j. But due to the factor $\exp(-i2kL)$ in $\eta(L)$, they contain terms which oscillate around zero with L. This L dependence can be removed approximately for weak disorder [14], or exactly by a variable transformation. Both alternatives are independent of the above approximations for large L. They can be applied to the original FPE as well. For weak disorder, the condition $kl_{nn'} \gg 1$ holds for all mean free paths. The smallest mean free path $l_{n_0n_0'}$ sets a characteristic distance over which the probability distribution changes. So if $kl_{n_0n_0'} \gg 1$ the L-dependent terms oscillate very often over this distance. This justifies the average over the oscillations (oscillating phase approximation). It is worth stressing that the validity of this approximation is not limited by the number of wires, in contrast to models with a fixed interwire coupling [14]. We note that the oscillating phase approximation leads to the same model as the weak disorder limit of the phenomenological model of Mello and Tomsovic [25] if their mean free paths $l_{nn'}$, $l'_{nn'}$ for backward and forward scattering are assumed to be identical. Thus the weak disorder limit of our model is a microscopic realization of a subclass of their model. For stronger disorder, the oscillating phase approximation is no longer valid. Instead, one can introduce the variable $u_{mn}\exp(ikL)$, which will be denoted by the same letter u_{mn} for notational convenience. The FPE in Eq. (6) is transformed to this variable by removing the factor $\exp(-i2kx)$ of $\eta_{nn'}$ and adding the term $-ik(u_{mn}\partial_{u_{mn}} - u^*_{mn}\partial_{u^*_{mn}})$ to $\hat{C}$. This is a systematic way to treat the oscillating terms, which yields the same results in the weak disorder limit but is exact. We emphasize that such a transformation can be found in general for all kinds of models.

Having removed the L dependence, we get a FPE which has a formal structure similar to the one in the work of Kree and Schmid for a strictly 1D wire [28]. For $N = 1$ their result is reproduced. We proceed analogously and introduce the characteristic function

$$P_j(L;\xi_j,\mathbf{u},\mathbf{u}^*) = \int d\Gamma_j \exp(i\xi_j\Gamma_j)p_j(L;\Gamma_j,\mathbf{u},\mathbf{u}^*) \quad (9)$$

for the moments of Γ_j,

$$\langle\Gamma_j^n\rangle_L = \int d\mu(\mathbf{u})(-i\partial_{\xi_j})^n P_j(L;\xi_j,\mathbf{u},\mathbf{u}^*)|_{\xi_j=0}. \quad (10)$$

At $L = 0$, $\mathbf{M} = \mathbf{1}$. Therefore, $\mathbf{\Gamma} = \mathbf{0}$ and $\mathbf{u}$ is arbitrary. For convenience, we choose the initial probability distribution $p_j(0;\Gamma_j,\mathbf{u},\mathbf{u}^*) = \delta(\Gamma_j)q_{st}(\mathbf{u},\mathbf{u}^*)$ where q_{st} is the stationary solution of the FPE $\hat{C}q_{st} = 0$. The characteristic function is given by

$$P_j(L;\xi_j,\mathbf{u},\mathbf{u}^*) = \exp\left\{(-\xi_j^2\hat{A}_j - i\xi_j\hat{B}_j + \hat{C})L\right\} q_{st}. \quad (11)$$

For the calculation of $\langle\Gamma_j\rangle$ we have to expand the exponential in Eq. (11) and to keep the terms $-i\xi_j\hat{C}^k\hat{B}_j\hat{C}^{k'}L^{k+k'+1}/(k+k'+1)!$ which are linear in ξ_j. Since $\hat{C}q_{st}$ is zero by definition and the integral $\int d\mu(\mathbf{u})\hat{C}f(\mathbf{u},\mathbf{u}^*)$ gives zero for any continuous complex function f on the unitary group [27], we obtain

$$\langle\Gamma_j\rangle_L = -L\int d\mu(\mathbf{u})\hat{B}_j q_{st}. \quad (12)$$

The evaluation of the first term of $\hat{B}_j$ in Eq. (12) leads to the expression for the Lyapunov exponents which was derived in [26]. Thus the second and third terms of $\hat{B}_j$ must not contribute to $\langle\Gamma_j\rangle_L$. We can prove this for the case of one wire and have checked it for some examples of N wires. Therefore we believe that it is true in general.

The *key problem* is the determination of q_{st}. The irreducible representations of the unitary group form a complete system. Their matrix elements are polynomials in u_{mn} and u^*_{mn} [29]. Therefore, q_{st} has an expansion of the form

$$q_{st}(\mathbf{u},\mathbf{u}^*) = \sum_{ab} c^{ab}_{m_1n_1\cdots m_an_a,m'_1n'_1\cdots m'_bn'_b} u_{m_1n_1}\cdots u_{m_an_a}u^*_{m'_1n'_1}\cdots u^*_{m'_bn'_b}. \quad (13)$$

Since one expects q_{st} to be unique, one can pursue the following strategy. Try to find a solution of $\hat{C}q_{st} = 0$ in the higher dimensional space of u_{mn} and u^*_{mn} which has the form given in Eq. (13). Then the property that different polynomials are linearly independent will lead to systems of linear equations for the expansion coefficients which have to be solved. It is not sure that a solution exists and it will not be unique since $\hat{C}f(\mathbf{uu}^\dagger)q_{st} = f(\mathbf{uu}^\dagger)\hat{C}q_{st}$ for any f. But if it exists, the restriction to the submanifold $\mathbf{uu}^\dagger = \mathbf{1}$ should be unique. Now we apply this to the case that $U_{nn'} = U/(N+1)$ so that $1/l_{nn'} = 1/l'_{nn'} = [1/l(N+1)](1+\delta_{nn'})$ and investigate the weak disorder limit. As indicated above, the weak disorder limit of our model is a special realization of the model of Mello and Tomsovic [25]. They termed the case in which the backscattering mean free paths have the above form ECM and proved that these models are equivalent to their former IM as far as averages over functions of $\mathbf{\Gamma}$ are concerned. We note that the choice $U_{nn'} = U/(N+1)$ represents a continuous 1D N-orbital model [30] coupled to ideal leads. This links the IM to the model of Iida, Weidenmüller, and Zuk where a discrete 1D N-orbital model was coupled (though in a different way) to ideal leads. A similar system has been already investigated in [31].

Assuming that q_{st} is unique, we will now *prove*, and this is our central result, that $q_{st} = 1$ for ECM's, i.e., $\mathbf{u}$ is uniformly distributed on the unitary group. The FP operator $\hat{C}$ has the form $k\hat{C}_0 + (1/l)\hat{C}_1$ where $\hat{C}_0 = -i(u_{mn}\partial_{u_{mn}} - u^*_{mn}\partial_{u^*_{mn}})$. Expanding q_{st} into $q_0 + (1/kl)q_1 + \cdots$ one obtains two equations involving q_0, $\hat{C}_0q_0 = 0$ and $\hat{C}_0q_1 + \hat{C}_1q_0 = 0$. The first implies that q_0 only contains polynomials with the same total degree in u_{mn} and u^*_{mn}. Let us denote the projector onto polynomials of this type by $\hat{P}_e$. Applying $\hat{P}_e$ to the second equation leads to $\hat{P}_e\hat{C}_1q_0 = \hat{P}_e\hat{C}_1\hat{P}_eq_0 = 0$. This defines q_0 completely. The point is that the terms in $\hat{C}_1$ which have no derivatives and which remain after the projection onto $\hat{P}_e\hat{C}_1\hat{P}_e$ only depend on the backscattering mean free paths and are zero if $\mathbf{uu}^\dagger = \mathbf{1}$. Removing these terms leads to an equivalent FP operator $\hat{C}_0 + (1/kl)\hat{C}'_1$ in the sense that the evolution on the submanifold $\mathbf{uu}^\dagger = \mathbf{1}$ is identical [27]. Now it is obvious that $q_0 = 1$ is a solution since $\hat{P}_e\hat{C}'_1\hat{P}_e$ only contains terms with derivatives with respect to u_{mn} and u^*_{mn}. This proves the assertion. Inserting $q_0 = 1$ into Eq. (12) leads to $\langle \Gamma_j \rangle_L = (2jL/l)/(N+1)$. Further results are obtained by applying the oscillating phase approximation to Eq. (6). Then $\hat{A}_j$ and the first term of $\hat{B}_j$ do not depend on $\mathbf{u}$ anymore. Integration over $\mathbf{u}$ leads to a closed FPE for Γ_j which is solved by a Gaussian probability distribution with the above mean and the variance $\mathrm{var}\{\Gamma_j\} = (4L/l)/(N+1)$. These results are in accordance with those of Macêdo and Chalker [24] for the IM as expected [25]. In addition, we find that Γ_j are Gaussian distributed.

In conclusion, the FPE describing quantum transport in disordered systems in the localized regime has been extended to samples of arbitrary transverse extension and disorder strength. Several connections between microscopic and phenomenological models have been established. An alternative method to calculate the limiting probability distribution of $\mathbf{u}$ has been applied to ECM's. We plan to apply it to models with three, four, or more wires in which only nearest-neighbor wires are coupled.

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